MODELING OF THE RHEOLOGICAL BEHAVIOR OF WOOD IN COMPRESSION PROCESSES

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A rheological equation of state for wood which allows for the running values of its porosity and humidity is considered.

A great number of methods for improving various properties of natural wood are known. During modification, wood samples can be subject to heating, drying, pressing, and resin or mineral-oil treatment. For example, in the thermomechanical method of modification of wood, bars are first dried at room temperature, with their humidity being brought to the level of 12-20%, then they are pressed in special molds and are dried again in the chambers at a temperature of about 110° C (the humidity of the samples is reduced in this manner to 10%); finally, they are allowed to cool in molds at room temperature [1]. Improvement of the technologies of obtaining modified wood requires theoretical investigation of all stages of the production process. The central problem in modeling the regimes of pressing is adequate description of the complex rheological behavior of wood.

The rheological properties of wood are determined by the special features of its anatomical and chemical structure. Wood cells form a complex capillary-porous structure. Changes of the stressed state and temperature-moisture effects cause transformations of the structure related to the special features of the dynamics of moisture in the system of pores and capillaries. The material itself of the walls of a wood cell is an intricate polymeric complex.

For rheological description of wood materials use was made of models allowing, in different ways, for their elastic, viscous, and plastic properties under stationary temperature-moisture conditions [1–4]. Employment of these equations for modeling the process of pressing is difficult due to continuous changes in the structure of the wood. Thus, in packing of the birch wood from the natural state with a density of ~560 kg/m³ to a density of 1100 kg/m³ the volumetric content of the cavities of the vessels and fibers of the libriform decreases in the sample from ~63 to 28% [2]. In this case, the humidity of the wood bar can increase 2–3 times [1]. It is known that a change of even several percent in the humidity, when the limit of saturation of the cell walls is not reached, substantially informs us about the mechanical behavior of wood. In modeling the processes of pressing under natural conditions, one must predict the evolution of the distributions of the porosity and initially nonuniform fields of humidity and temperature over the entire volume of the sample.

By virtue of this, it is topical to construct phenomenological models which allow for the special features of the capillary-porous structure and the multiphase character of wood. In this work, we have formulated a rheological model which takes into account the running values of its humidity and porosity.

Wood is a three-phase system consisting of the woody substance, water which partially fills pores and capillaries, and a vapor-gas mixture. One can obtain continuum equations for the heterophase medium by volumetric averaging of microequations for macroscopic parameters of each phase [5], proceeding from the methods of the mechanics of multiphase media [6]. We note that this approach has not been applied earlier to study the processes of modification of wood.

Assuming that pulsed transfer of momentum and energy in all phases can be neglected, we can represent the total stress tensor of the considered heterogeneous system as a sum of the averaged stresses in the phases:

$$\sigma^{kl} = \alpha_1 \langle \sigma_1^{kl} \rangle_1 + \alpha_2 \langle \sigma_2^{kl} \rangle_2 + \alpha_3 \langle \sigma_3^{kl} \rangle_3.$$
⁽¹⁾

If the deformations of microvolumes are small, the microdeformation tensor in the solid phase can be written as

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$$\varepsilon_{3}^{ij} = \frac{1}{2} \left(\frac{\partial h_{3}^{ij}}{\partial x^{i}} + \frac{\partial h_{3}^{i}}{\partial x^{j}} \right).$$

After averaging it can be represented in the following form:

$$\langle \boldsymbol{\varepsilon}_{3}^{jj} \rangle_{3} = \boldsymbol{\varepsilon}_{3}^{ij} - \boldsymbol{\varepsilon}_{\mathrm{f}}^{ij} \,. \tag{2}$$

Here ε_3^{ij} is determined by the gradients of the mean displacements of the material, $\varepsilon_3^{ij} = \frac{1}{2} \left(\frac{\partial \langle h_3^i \rangle_3}{\partial' x^i} + \frac{\partial \langle h_3^i \rangle_3}{\partial' x^j} \right) \varepsilon_f^{ij}$ is the fictitious or effective tensor of deformations, which characterizes the displacement of the elements of the structure skeleton [6]:

$$\varepsilon_{\rm f}^{ij} = -\frac{1}{2} \left[\langle h_3^{,j} \rangle_3 \frac{\nabla^i \alpha_3}{\alpha_3} + \langle h_3^{,j} \rangle_3 \frac{\nabla^j \alpha_3}{\alpha_3} \right] - \frac{s_{13}}{2\alpha_3} \langle h_3^{,j} n_3^{,j} + h_3^{,i} n_3^{,j} \rangle_{13} + \frac{s_{23}}{2\alpha_3} \langle h_3^{,j} n_3^{,j} + h_3^{,i} n_3^{,j} \rangle_{23}, \quad s_{ij} = \frac{dS_{ij}}{dV},$$

i.e., the observed macrodeformations of the solid phase ε_3^{ij} are composed of the deformations of the wood substance $\langle \varepsilon_3^{\prime ij} \rangle_3$ and the deformations of the wood skeleton ε_f^{ij} , which lead to restructurization of the system of pores.

Following [6], we adopt the macroscopic hypothesis on the rheological behavior separately for the solid-phase material and the structural skeleton of the system.

Allowing for the fact that wood cells are of polymeric nature, it is natural to take for description of the stressed-deformed state of the wood material one of the models of a hereditary anisotropic medium [7]:

$$\langle \boldsymbol{\varepsilon}_{3}^{'ij} \rangle_{3} = \Pi_{s}^{ijkl} (0) \left[\langle \boldsymbol{\sigma}_{3}^{'kl} \rangle_{3} + \int_{0}^{t} K_{s}^{ijkl} (t-\tau) \langle \boldsymbol{\sigma}_{3}^{'ij} \rangle_{3} d\tau \right] + \Lambda_{s}^{ij} \langle \boldsymbol{\Theta}_{3}^{'} \rangle_{3} .$$

$$(3)$$

To fictitious deformations there must correspond fictitious stresses. We construct the tensor of the latter in the wood skeleton. For this purpose we consider stresses in the wood as in the medium with dual porosity and as in the composite material representing a porous matrix (wood substance pierced by capillaries) and a system of macropores in it. Heat and mass in the softwood are transferred, on the one hand, through the system of tracheids ($a \sim 20-50 \,\mu\text{m}$), woodrays ($a \sim 10 \,\mu\text{m}$), bordered and simple pits ($a \sim 1-10 \,\mu\text{m}$) and, on the other hand, through microcapillaries in the cell walls [8]. Similarly, in the hard wood we distinguish the system of macropores to which vessels ($a \sim 30-400 \,\mu\text{m}$), fibers of the libriform ($a \sim 15-20 \,\mu\text{m}$), and woodrays are referred. The capillaries appear in the cell wall with a first portion of sorbed water, which disperses the wood substance, thus transferring it to the colloid state [9]. In a dry cell wall, they are virtually absent. The diameter of the capillaries increases as the humidity changes and it is $a \sim 0.005-0.040 \,\mu\text{m}$ [8]. Thus, we can treat wood as two porous media, which are enclosed into each other and possess intrinsic porosity, permeability, and other structural mechanical properties, and use the investigation methods developed for these systems [10, 11].

Following [11], the total averaged stresses can be represented in the form

$$\sigma^{kl} = (1 - m_{\rm p}) \langle \sigma^{\prime kl}_{\rm cell} \rangle_{\rm cell} + m_{\rm p} \langle \sigma^{\prime kl}_{\rm p} \rangle_{\rm p} , \qquad (4)$$

where $\langle \sigma_{cell}^{\prime kl} \rangle_{cell}$ are the averaged stresses in the cell wall pierced by capillaries and $\langle \sigma_p^{\prime kl} \rangle_p$ are the mean stresses in the micropores.

The total averaged stresses in the cell walls are written as

$$\langle \sigma_{\text{cell}}^{kl} \rangle_{\text{cell}} = (1 - m_{\text{c.s}}) \langle \sigma_3^{kl} \rangle_3 + m_{\text{c.s}} \langle \sigma_{\text{c.s}}^{kl} \rangle_{\text{c.s}}, \qquad (5)$$

where $\langle \sigma_{c,s}^{\prime kl} \rangle_{c,s}$ are the mean stresses in the capillaries of the cell walls and $\langle \sigma_3^{\prime kl} \rangle_3$ are the mean stresses in the wood skeleton.

We denote the volumetric content of the vapor-gas mixture and the liquid in large pores by α_{p1} and α_{p2} and in the capillaries by $\alpha_{c.s1}$ and $\alpha_{c.s2}$; then

$$\begin{split} \langle \sigma_{p}^{kl} \rangle_{p} &= \alpha_{p1} \langle \sigma_{1}^{kl} \rangle_{1} + \alpha_{p2} \langle \sigma_{2}^{kl} \rangle_{2}, \quad \langle \sigma_{c,s}^{kl} \rangle_{c,s} = \alpha_{c,s1} \langle \sigma_{1}^{kl} \rangle_{1} + \alpha_{c,s2} \langle \sigma_{2}^{kl} \rangle_{2}, \\ \alpha_{p1} + \alpha_{p2} = 1, \quad \alpha_{c,s1} + \alpha_{c,s2} = 1. \end{split}$$

Assuming that the gas is ideal and it is sufficient to allow for the viscosity of the liquid only in interphase

interactions, we have $\langle \sigma_1^{kl} \rangle_1 = -p_1 \delta^{kl}$ and $\langle \sigma_2^{\prime kl} \rangle_2 = -p_2 \delta^{kl}$, $p_2 = p_1 + p_c$. The tensor of fictitious stresses for the porous system whose porosity is determined by macropores only, can be calculated, as in [11], by the expression $(1 - m_p) [\langle \sigma_{cell}^{\prime kl} \rangle_{cell} - \langle \sigma_p^{\prime kl} \rangle_p]$.

Using (5) and (4), we introduce the tensor of fictitious stresses related to the transfer of forces between the structural elements of the skeleton:

$$\sigma_{\rm f}^{kl} = (1 - m_{\rm c.s}) \left[\langle \sigma_3^{kl} \rangle_3 - \langle \sigma_{\rm c.s}^{kl} \rangle_{\rm c.s} \right] = \langle \sigma_{\rm cell}^{kl} \rangle_{\rm cell} - \langle \sigma_{\rm c.s}^{kl} \rangle_{\rm c.s} = \frac{\sigma^{kl} - m_{\rm p} \langle \sigma_{\rm p}^{kl} \rangle_{\rm p}}{1 - m_{\rm p}} - \langle \sigma_{\rm c.s}^{kl} \rangle_{\rm c.s} \,. \tag{6}$$

After transformations of (6), allowing for $\alpha_1 = \alpha_{p1}m_p + \alpha_{c.s1}m_{c.s}$ and $\alpha_2 = \alpha_{p2}m_p + \alpha_{c.s2}m_{c.s}$, we can obtain the expression which generalizes the equation, similar to that in [11], to the case of unsaturated media:

$$\sigma_{\rm f}^{kl} = \frac{\sigma^{kl}}{1 - m_{\rm p}} + \left[\frac{m_{\rm p}\alpha_{\rm p1}}{1 - m_{\rm p}} + \alpha_{\rm c.s1}\right] p_1 \delta^{kl} + \left[\frac{m_{\rm p}\alpha_{\rm p2}}{1 - m_{\rm p}} + \alpha_{\rm c.s2}\right] p_2 \delta^{kl} \,. \tag{7}$$

If $m_p < 1$, i.e., the concentration of macropores is low, $\sigma_f^{kl} \approx \sigma^{kl} + \alpha_{c,s1} p_1 \delta^{kl} + \alpha_{c,s2} p_2 \delta^{kl}$. In the case where the capillaries are virtually filled with moisture ($\alpha_{c,s1} \sim 0$, $\alpha_{c,s2} \sim 1$), we have $\sigma_f^{kl} \approx \sigma^{kl} - p_2 \delta^{kl}$, i.e., the tensor of fictitious stresses is determined in the same manner as for saturated cracked-porous media [6, 10, 11]. To the particular case of the state of wood with a small content of liquid in macropores and the vapor-gas mixture in capillaries ($\alpha_{p2} \sim 0$, $\alpha_{c,s} \sim 0$) there correspond fictitious stresses of the form

$$\sigma_{\rm f}^{kl} = \frac{\sigma^{kl}}{1 - m_{\rm p}} + \frac{m_{\rm p}}{1 - m_{\rm p}} p_1 \delta^{kl} + p_2 \delta^{kl} \,. \tag{8}$$

Following [6], we assume that the dependence of the tensor of fictitious stresses in the solid phase on the effective tensor of deformations is similar to the rheological equation for the material of the wood skeleton:

$$\varepsilon_{\rm f}^{ij} = \Pi_{\rm f}^{ijkl} \left(0\right) \left[\sigma_{\rm f}^{kl} + \int_{0}^{t} K_{\rm f}^{ijkl} \left(t - \tau\right) \sigma_{\rm f}^{ij} d\tau \right].$$
(9)

Equations (1)-(3), (7), and (9) determine the stressed-deformed state in the wood. The dependence of the tensor of total stresses in the wood on macrodeformations can be obtained from the above-given relations:

$$\begin{split} \varepsilon^{ij} &= \Pi_{\rm s}^{ijkl} \left(0 \right) \left[\frac{\sigma^{kl} + \alpha_2 p_2 \delta^{kl} + \alpha_1 p_1 \delta^{kl}}{\alpha_3} + \int_0^t K_{\rm s}^{ijkl} \left(t - \tau \right) \frac{\sigma^{ij} + \alpha_2 p_2 \delta^{ij} + \alpha_1 p_1 \delta^{ij}}{\alpha_3} d\tau \right] + \Lambda_{\rm s}^{ij} \Theta_3 + \\ &+ \Pi_{\rm f}^{ijkl} \left(0 \right) \left[\frac{\sigma^{kl}}{1 - m_{\rm p}} + \beta_2 p_2 \delta^{kl} + \beta_1 p_1 \delta^{kl} + \int_0^t K_{\rm f}^{ijkl} \left(t - \tau \right) \left(\frac{\sigma^{ij}}{1 - m_{\rm p}} + \beta_2 p_2 \delta^{ij} + \beta_1 p_1 \delta^{ij} \right) d\tau \right], \end{split}$$

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$$\beta_1 = \frac{m_p \alpha_{p1}}{1 - m_p} + \alpha_{c.s1}; \quad \beta_2 = \frac{m_p \alpha_{p2}}{1 - m_p} + \alpha_{c.s2}.$$

The rheological equation (10) involves explicitly the characteristics of humidity and porosity of the material. We note that it can be generalized to the case where the wood substance and the wood skeleton do not possess the property of the linearity of deformations.

We consider certain parameters involved in (10). We can estimate the range of variation of the coefficient β_1 if we approximately take the air content of the pine wood equal to 69, 66, and 32% at a humidity of 0, 12, and 100%, respectively, as the volumetric content of large pores [8]. The value of β_1 cannot exceed 2.3. Allowing for this and also for the fact that $\alpha_2 < 1$ and virtually realizable compression stresses are from 20 to 50 MPa, in (10) we can neglect all the terms containing p_1 (pressure of the vapor-gas mixture is close to atmospheric).

The capillaries of cell walls are virtually completely filled with water (in the absence of water they collapse); therefore, $\alpha_{c.s1} \sim 0$, $\alpha_{c.s2} \sim 1$ and $1 - m_p \approx \alpha_3 + u\alpha_2$; $u = \gamma/(\gamma + 1)$. The parameter γ is equal to the ratio of the volume of water in the capillaries to the volume of water in the system of pores. The range of variation of γ is not bounded from above. We have the value $\gamma \sim \infty$ is for absolutely dry ($w \sim 0\%$) and room-dry ($w \sim 8-12\%$) wood. For the wood close to the state of fiber saturation a rough estimate gives $\gamma \sim 0.01$. This estimate allows for the fact that at $w \sim 30\%$ the films of adsorption water are present on the surface of macropores. The coefficient β_2 can be represented as $\beta_2 = 1 + \alpha_2/[(\alpha_3 + u\alpha_2)(\gamma + 1)]$. Consequently, $1 < \beta_2 < 2$ and the terms containing capillary pressure can be disregarded in (10) when the quantity p_2 is much smaller than the compression stresses.

The value of the capillary pressure greatly depends on the degree of saturation of the porous medium, which in turn changes with deformation. For determination of the capillary pressure we employ the Aver'yanov formula used for calculating this quantity in grounds [12]:

$$p_{\rm c}(\theta) = -p_0 \frac{\theta_0}{\theta} \frac{1-\theta^3}{1-\theta_0^3}, \quad \theta = \frac{\alpha_2}{\alpha_2 + \alpha_1}.$$
(11)

Here p_0 the capillary pressure at the saturation θ_0 in the case of moistening of the porous medium only with adsorption water. The sorption abilities of the wood substance are such that the absorption of moisture by polymolecular adsorption is predominant at a relative humidity of air from 0.2 to 0.7 and a humidity of the wood no lower than 5% [8]. In calculations, we can take the value of saturation θ_0 corresponding to a humidity of the wood of ~6%. We note that formula (11) is obtained for the porous system with a stationary system of pores and capillaries. In the wood, the capillary structure experiences changes with increase in the humidity; by virtue of this the quantity $p_0(\theta_0)$ refers, strictly speaking, to a somewhat different porous structure as compared to the quantity $p_2(\theta)$. In calculations, it is taken that

$$p_0 = \frac{2\Sigma}{R_{\rm char}} C$$

where $C = C(\theta)$ is a certain saturation function and R_{char} is the characteristic radius of microcapillaries of this type of wood ($R_{char} < 10^{-7}$ m).

Experimental data must be used for determining the parameters of the functions of the creep rate and the values of the instantaneous compliance. We study the possibility of application of the constructed rheological model to approximation of the experimental results [4].

Of the large series of experiments performed in [4], we consider the processes of creep in stepwise variation of the compression stress along the fibers of the pine wood under stationary temperature conditions. The experiments were made in one-axial loading following the conditions providing the uniformity of deformations. The experimental data are given in [4] in the form of graphs for different values of the humidity of the samples and the compression stresses.

In the deformation, the volume of the wood and, consequently, the values of the content of all phases change, which makes calculations of the creep functions difficult. For one-dimensional and homogeneous deformations we have

$$\alpha_i = \alpha_{i0} \frac{V_0}{V} = \alpha_{i0} \frac{H_0}{H} + \alpha_{i0} \frac{1}{1 - \epsilon}, \quad i = 1, 2, 3.$$
(12)

Substituting the law of stepwise variation of stress $\sigma = \sigma \cdot 1(t)$ (1(*t*) is the Heaviside function) into (10), neglecting the effect of the pressure of the vapor-gas phase, and using (12), we obtain the integral equation for determining the deformation, which contains the parameters dependent now on the initial values of the porosity and the humidity of the wood. Allowing for the possible nonlinearity of the creep, under stationary temperature conditions we have

$$\varepsilon = I_{s0} \left[F_{s} \left(\frac{\sigma \left(1 - \varepsilon \right) + \alpha_{20} p_{2}}{\alpha_{30}} \right) + \int_{0}^{t} \kappa_{s} \left(t - \tau \right) F_{s} \left(\frac{\sigma \left(1 - \varepsilon \right) + \alpha_{20} p_{2}}{\alpha_{30}} \right) d\tau \right] + I_{f0} \left[F_{f} \left(\frac{\sigma \left(1 - \varepsilon \right) + \left(\alpha_{30} + \alpha_{20} \right) p_{2}}{\alpha_{30} + u \alpha_{20}} \right) + \int_{0}^{t} \kappa_{f} \left(t - \tau \right) F_{f} \left(\frac{\sigma \left(1 - \varepsilon \right) + \left(\alpha_{30} + \alpha_{20} \right) p_{2}}{\alpha_{30} + u \alpha_{20}} \right) d\tau \right],$$
(13)

$$p_{2} = -p_{0} \frac{\alpha_{2} |_{w=6\%}}{\alpha_{20}} (1-\varepsilon) \frac{(1-\varepsilon-\alpha_{30})^{3} - \alpha_{20}^{3}}{(1-\varepsilon-\alpha_{30})^{3} - (1-\varepsilon)^{3} (\alpha_{2} |_{w=6\%})^{3}}.$$
 (14)

For description of the deformation the exponential influence functions and weakly singular kernels of the Koltunov creep are used [7]:

$$\kappa_{j}(t) = d_{j1} \exp\left(-t/\lambda_{j1}\right), \quad \kappa_{j} = \frac{\exp\left(-\eta_{j}t\right)}{t} \sum_{n=1}^{\infty} \frac{\left[A_{j} \Gamma\left(\omega_{j}\right)\right]^{n} t^{\omega_{j}n}}{\Gamma\left(\omega_{j}n\right)}, \quad j = s, f.$$
(15)

The stress function is selected in the form of the cubic dependence

$$F_j(\sigma) = b_{j1}\sigma + b_{j2}\sigma^3, \quad j = s, f.$$
(16)

The unknown parameters of the rheological model were determined from the condition of the minimum of the function which for the first type of influence functions has the form

$$\Phi(I_{s0}, I_{f0}, d_{s1}, \lambda_{s1}, d_{f1}, \lambda_{f1}, b_{s1}, b_{s2}, b_{f1}, b_{f2}, u, C) = \sum_{k=0}^{N} \left[\frac{\varepsilon_{\exp}(t_k) - \varepsilon_{\text{theor}}(t_k)}{\varepsilon_{\exp}(t_N)} \right]^2.$$
(17)

For the second type of influence functions the minimized function is constructed similarly. All the calculations have been done with the help of a computer.

For calculations we need data on the initial volumetric content of the solid phase α_{30} , moisture α_{20} , and the vapor-gas mixture α_{10} in the test samples. Given the initial values of humidity and porosity of the sample (they change during compression due to the change in the structure of the pores), we can find these quantities from the following system of equations:

$$\alpha_{10} + \alpha_{20} + \alpha_{30} = 1, \quad \rho = \rho_1 \alpha_{10} + \rho_2 \alpha_{20} + \rho_3 \alpha_{30}, \quad \frac{w}{100} = \frac{\rho_2}{\rho_3} \frac{\alpha_{20}}{\alpha_{30}} + \frac{\rho_1}{\rho_3} \frac{\alpha_{10}}{\alpha_{30}}.$$
 (18)

We note that more accurate calculation requires replacement, in the last equation of (18), of the volumetric content of the vapor-gas mixture by the value of the volume concentration of pure water vapor.

Absolute humidity, %	Volumetric content		
	solid phase	water	vapor-gas mixture
6	0.298	0.027	0.675
10	0.291	0.044	0.665
15	0.284	0.064	0.652
20	0.278	0.084	0.638
40	0.264	0.161	0.575

TABLE 1. Calculated Values of the Initial Volumetric Content of the Phases in the Pine-Wood Sample



Fig. 1. Curves of the pine-wood creep in compression along the fibers at different values of the initial humidity of the sample (a) [1) w = 6%, 2) 10, 3) 15, and 4) 20 ($-\sigma = 9.81$ MPa)] and different levels of stress (b) [1) $-\sigma = 9.81$ MPa, 2) 19.62, and 3) 32.57; w = 10%]. Solid lines, theoretical calculation; points, experimental data [4]. *t*, months.

If the density of the sample ρ is unknown (the values of the densities of the tested samples are not given in [4]), we can find it from the humidity using the known empirical formulas [8]. Moreover, approximating the test values of the air content of the wood (for the pine wood these data are given in [8]), we obtain an approximate relation which can be used instead of the second equation of (18):

$$\alpha_{10} = 0.069 - 0.234 (w/100) - 0.1361 (w/100)^2$$
.

The calculated values of the volumetric content of each of the three phases in the pine-wood sample before deformation at different humidities are given in Table 1.

The computer calculations allowed determination of the parameters of the one-dimensional model (13) for both influence functions (15). The sum of the squares of deviations of the calculated values from experimental ones is less than unity in all the cases. The optimum values of the rheological parameters for the creep kernels in the form of exponents are as follows: $I_{s0} = 3.058 \cdot 10^{-2} \text{ Pa}^{-1}$, $I_{f0} = 4.179 \cdot 10^{-12} \text{ Pa}^{-1}$, $d_{s1} = 2.711 \cdot 10^{-6} \text{ sec}^{-1}$, $d_{f1} = 5.992 \cdot 10^{-6} \text{ sec}^{-1}$, $\lambda_{s1} = 2.177 \cdot 10^5 \text{ sec}$, $\lambda_{f1} = 3.883 \cdot 10^5 \text{ sec}$, $b_{s1} = 1.184$, $b_{s2} = 7.222 \cdot 10^{-17} \text{ Pa}^{-1}$, $b_{f1} = 1.506$, and $b_{f2} = -3.029 \cdot 10^{-18} \text{ Pa}^{-2}$. The rheological parameters of the equation with a Koltunov creep kernel are $I_{s0} = 1.529 \cdot 10^{-2} \text{ Pa}^{-1}$, $I_{f0} = 1.478 \cdot 10^{-11} \text{ Pa}^{-1}$, $\omega_s = \omega_f = 0.06$, $A_s = 3.972 \cdot 10^5$, $A_f = 8.085 \cdot 10^{-5}$, $\eta_s = 1.157 \cdot 10^{-6} \text{ sec}^{-1}$, $\eta_f = 1.543 \cdot 10^{-6} \text{ sec}^{-1}$, $b_{s1} = 1.322$, $b_{s2} = 3.637 \cdot 10^{-17} \text{ Pa}^{-2}$, $b_{f1} = 1.015$, and $b_{f2} = 1.559 \cdot 10^{-17} \text{ Pa}^{-2}$. The coefficient of surface tension is $\Sigma = 0.075 \text{ kg/sec}^2$, and the characteristic radius of the capillaries is $R_{char} = 10^{-7} \text{ m}$. The fitting parameters for the capillary pressure and the volumetric content of macropores for both versions of the functions of the creep rate are taken to be the same: C = 0.5, 9, 14, and 22 and u = 1, 1, 0.3, and 0.1 at w = 6, 10, 15, and 20\%, respectively.

Figure 1 shows the curves of deformation changes in loading along the fibers, which are obtained for the pine wood both experimentally and by the suggested nonlinear hereditary rheological model with Koltunov influence functions. Satisfactory approximation of experimental data on the creep for the considered cases of loading under the conditions where the stresses do not exceed the limit of durable resistance ($-\sigma_{Ld,s} = 32.57$ MPa) and the initial humidity

of the samples varies from 6 to 20% is illustrated. We note that the choice of the kernel tells little about the mathematical accuracy of the approximation of the experimental data. However, use only of the singular kernel, which does not allow the jump of the derivative function of the creep rate on transition of the elastic deformations to viscoelastic ones, is physically justified [7].

Thus, the rheological equation (10) and its generalization to the nonlinear region of deformation can be used for theoretical investigation of the processes of aftereffect in the wood with allowance for the change in its porous structure and moisture content.

NOTATION

 n_3 , normal to the phase interface (external relative to phase 3); $\langle ... \rangle_i$, averaging over the volume of the phase *i*; $\langle ... \rangle_{ii}$, averaging of the interface between the phases *i* and *j*; *dV*, elementary macrovolume, m³; *dS*_{ii}, surface area of the interface between the phases i and j inside dV, m²; x, coordinate; s_{ij} , specific surface area of the interface between the phases i and j, m^{-1} ; α , volumetric content of the phase; w, humidity, %; θ , saturation; Σ , coefficient of surface tension, kg/sec²; p, pressure, Pa; σ^{kl} , stress tensor, Pa; σ , compression stress along the fibers, Pa; ϵ^{kl} , deformation tensor; ε , compression deformation along the fibers; K^{ijkl} , tensor of the functions of the creep rate; Λ^{kl} , tensor of the coefficients of temperature expansion, K^{-1} ; A, ω , and η , parameters of the Koltunov influence functions; $\Pi^{ijkl}(0)$, tensor of instantaneous compliances, Pa^{-1} ; I_{i0} (j = s, f), instantaneous compliance along the fibers, Pa^{-1} ; κ , function of the creep rate along the fibers; Θ , difference between the current temperature and some initial value of it, K; ρ , density, kg/m³; a, characteristic dimension of the pores, m; h, displacement, m; H, sample height, m; V, sample volume, m³; $m_{\rm p}$, volumetric content of large pores in the wood material; $m_{\rm c.s.}$, volumetric content of capillaries in the cell walls; F, stress function; Γ , Euler gamma function; δ^{kl} , unit tensor; R, radius, m; t, time, sec; τ , time preceding the time of observation t, sec; b_{i1} and b_{i2} , parameters of the cubic stress function; d and λ , parameters of the exponential influence function; sec⁻¹ and sec; Φ , minimized function; ∇ , differential operator; $u = \gamma/(\gamma + 1)$; $\gamma = \alpha_{c,s}/\alpha_{p,2}$. Subscripts: 1, 2, and 3, vapor, water, and solid phase; i, j, phase number; f, effective (fictitious); s, wood substance; p, system of macropores; cell, cell walls; c.s, capillary system; 0, initial; c, capillary; l.d.s, limit of durable resistance; exp, experimental; theor, theoretical (values of deformations which satisfy the rheological equation (13)); char, characteristic; N, the last point of the time interval. Superscripts: i, j, k, l, tensor components; ', parameters which are mean within the microvolume $d'V \ll a^3$.

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